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Most turbulent flows cannot be computed directly from the Navier-Stokes equations, because the convective term $\mathcal{C}(\boldsymbol{u},\boldsymbol{v}) = (\boldsymbol{u}\cdot\nabla)\boldsymbol{v}$ produces too many scales of motion. In quest of a dynamically less complex mathematical formulation, we consider regularizations $\mathcal{C}_n(\boldsymbol{u},\boldsymbol{v})$ of the nonlinearity. Hereby, the regularizations conserve the energy, enstrophy (2D) and helicity by construction. This yields $\mathcal{C}_n = \mathcal{C} + \mathcal{O}(\epsilon^n)$, where ϵ is the filter length, and $n = 2, 4, 6$. The regularized system is more amenable to approximate numerically, while its solution approximates the large-scale dynamical behavior of the Navier-Stokes solution. The evolution of the vorticity $\boldsymbol{\omega}$ is given by the usual equation, where the vortex stretching term becomes $\mathcal{C}_n(\boldsymbol{\omega},\boldsymbol{u})$. By analyzing the regularized interactions, the filter length is determined such that the production of smaller sales by means of vortex stretching stops at the grid-size δ.
As a first step, the method is applied to Burgers' equation with $\mathrm{Re} = 50$. Here, the analysis is relatively easy, while important aspects of 3D Navier-Stokes remain. To illustrate the results, energy spectra are shown below for $n = 4$. Clearly, without the model the physics are not captured correctly, as the energy is not dissipated enough at the high wavenumbers. With the model, a power law of k^{-20} for small scales is found, for different values of δ.

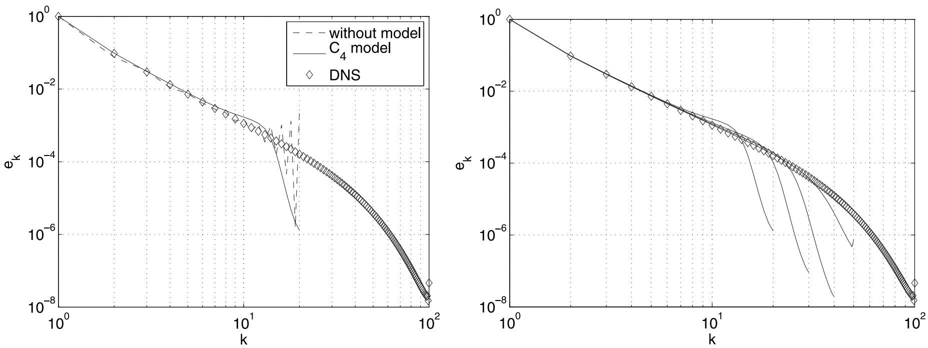

Fig. 1. *Left: Steady state energy spectra, with and without the model. Right: Steady state energy spectra for different values of the grid-size δ.*